

LIMITS OF ACCURACY

(UPPER/LOWER BOUND)

$$A = 10 \text{ km} \div 2 \quad \text{Error} = 5 \text{ km}$$

(3 Marks) (P1)

Distance of Lahore to Islamabad is 400 km to nearest 10 km

Time taken to travel is 4 hours to nearest hour

$$A = 1 \text{ h} \\ \div 2 \\ E = 0.5 \text{ h}$$

(i)		LOWER BOUND	UPPER BOUND
	DISTANCE	$\text{orig} - E$ $400 - 5$ $= 395$	$\text{orig} + E$ $400 + 5$ $= 405$
	TIME	$\text{orig} - E$ $4 - 0.5$ $= 3.5 \text{ h}$	$\text{orig} + E$ $4 + 0.5$ $= 4.5 \text{ h}$

$$\text{ERROR} = \frac{\text{ACCURACY}}{2}$$

$$\begin{aligned} \text{UPPER BOUND} &= \text{ORIG} + \text{ERROR} \\ \text{LOWER BOUND} &= \text{ORIG} - \text{ERROR} \end{aligned}$$



(ii) Find upper bound of speed → indirect → orig value missing
→ accuracy missing

$$\uparrow \text{Speed} = \frac{\uparrow \text{Distance}}{\downarrow \text{Time}} = \frac{405}{3.5} = \boxed{}$$

(iii) Find lower bound of speed

$$\downarrow \text{Speed} = \frac{\downarrow \text{Distance}}{\uparrow \text{Time}} = \frac{395}{4.5} = \boxed{}$$

Advanced: INDIRECT

Sometimes question will ask for upper bound or lower bound of an indirect value whose original value or accuracy is not given in question.

STEP 1: Write formula for the required quantity.

STEP 2: Analyse the arrows on the equation. First arrow is always given in question.

ADD / SUBTRACT

+ve term = Same arrow

-ve term = opp arrow

$$\uparrow A = \uparrow B + \uparrow C$$

$$\downarrow A = \downarrow B + \downarrow C$$

$$\uparrow A = \uparrow B - \downarrow C$$

$$\downarrow A = \downarrow B - \uparrow C$$

MULTIPLY / DIVIDE

Numerator = Same arrow

Denominator = opp arrow.

$$\uparrow A = \uparrow B \times \uparrow C$$

$$\downarrow A = \downarrow B \times \downarrow C$$

$$\uparrow A = \frac{\uparrow B}{\downarrow C}$$

$$\downarrow A = \frac{\downarrow B}{\uparrow C}$$

Q: Mass of a box is 20kg to nearest 4kg
 Mass of its packing is 2kg to nearest tenth of a kg.

$$A = 0.1\text{kg}$$

$$E = 0.05\text{kg}$$

	Lower Bound	Upper Bound
Box	$20-2$ 18	$20+2$ 22
Packing	$2-0.05$ 1.95	$2+0.05$ 2.05

(ii) Find the upper bound of total mass of box and its packing.

$$\begin{aligned}\uparrow \text{Total} &= \uparrow \text{Box} + \uparrow \text{Packing} \\ &= 22 + 2.05 \\ &= 24.05\end{aligned}$$

TEN	10	0.1	TENTH
HUNDRED	100	0.01	HUNDREDTH
THOUSAND	1000	0.001	THOUSANDTH

- 2 The cost of a chair, \$x, is \$70 correct to the nearest \$10. $A=10, E=5$
 The cost of a desk, \$y, is \$900 correct to the nearest \$50. $A=50, E=25$

(a) Complete the table in the answer space.

(b) Find the lower bound of the total cost of a desk and 4 chairs.

$$\begin{aligned} \text{Total} &= 1 \text{ Desk} + 4 \text{ chairs} \\ \downarrow \text{Total} &= \downarrow y + 4x \downarrow \\ &= 875 + 4(65) \\ &= 1135 \end{aligned}$$

Answer (a)

	Lower Bound	Upper Bound
Chair x	65	75
Desk y	875	925

[2]

(b) \$..... [1]

- 6 In a race, an athlete runs 1600 m at an average speed of 6 m/s.
 (The distance is given correct to the nearest 100 m and the speed correct to the nearest metre per second.)

(a) Complete the two statements in the answer space.

(b) Calculate the greatest possible time the race could have taken.

Distance

orig = 1600m

A = 100m

E = 50m

Speed

orig = 6 m/s

A = 1

E = 0.5

Time (indirect)

$$\uparrow t = \frac{d \uparrow}{s \downarrow} = \frac{1650 \times 10}{5.5 \times 10} = \frac{16500}{55} = 300$$

LB

UB

Answer (a) 1550 ≤ distance < 1650

5.5 ≤ speed < 6.5 [2]

(b) 300 seconds [1]

- 3 (a) The population of a city is given as 280 000, correct to the nearest ten thousand.
State the greatest possible error in the given value.
- (b) The dimensions of a rectangular card are 7 cm by 4 cm, correct to the nearest centimetre.
Calculate the smallest possible perimeter of the card.

Answer (a) [1]

(b)cm [1]

- 4 (a) A rectangular table top is 100 cm long and 75 cm wide.
Both lengths are correct to the nearest 5 cm. $A=5, E=2.5$
Find the least possible perimeter of the table.

- (b) The area of a rectangular room is 22 m^2 , correct to the nearest square metre. $A=1, E=0.5$
The width is 3 m, correct to the nearest metre. $A=1, E=0.5$
Find the greatest possible length of the room.

$A=1, E=0.5$



(i) Length = 100 cm $\begin{cases} UB = 102.5 \\ LB = 97.5 \end{cases}$
Width = 75 cm $\begin{cases} UB = 77.5 \\ LB = 72.5 \end{cases}$

$$\downarrow P = 2L \downarrow + 2w \downarrow$$

$$= 2(72.5) + 2(97.5)$$

$$= 340$$

(ii) Area = 22 m^2 $\begin{cases} UB = 22.5 \\ LB = 21.5 \end{cases}$
Width = 3 m $\begin{cases} UB = 3.5 \\ LB = 2.5 \end{cases}$

$$\uparrow L = \frac{\uparrow A}{\downarrow w} = \frac{22.5}{2.5}$$

$$= \frac{225}{25} = 9$$

~~$A = L \times W$~~

Answer (a)cm [2]

(b)m [2]

- 11 The length of a side of an equilateral triangle is given as 41 mm, correct to the nearest millimetre.

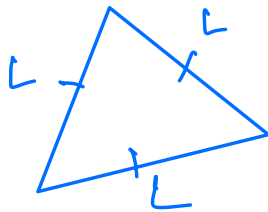
(a) Write down the lower bound for the length of a side.

$$A=1$$

$$E=0.5$$

Answer (a) 40.5 mm [1]

(b) Giving your answer in **centimetres**, calculate the lower bound for the perimeter of the triangle.



$$\downarrow P = \downarrow L + \downarrow L + \downarrow L$$

$$= 40.5 + 40.5 + 40.5$$

$$= 121.5 \text{ mm}$$

$$\begin{array}{r} \div 10 \\ 12.15 \end{array}$$

Answer (b) 12.15 cm [1]

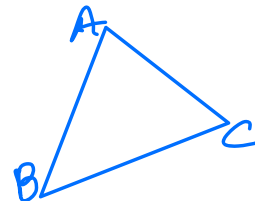
- 12 ABC is a triangle.

Angle A is 62° , correct to the nearest degree. $A=1, E=0.5$

Angle B is 53.4° , correct to the nearest tenth of a degree. $A=0.1, E=0.05$

(a) Write down the lower bound for angle B .

$$53.4 - 0.05 = 53.35$$



Answer (a) [1]

(b) Calculate the upper bound for angle C .

$$\uparrow C = 180 - \downarrow A - \downarrow B$$

$$= 180 - 61.5 - 53.35$$

$$= \boxed{}$$

Answer (b) [2]